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SUSTAINABLE ECONOMIC GROWTH: STRUCTURAL TRANSFORMATION WITH CONSUMPTION FLEXIBILITY

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Sustainable Economic Growth:

Structural Transformation with Consumption Flexibility

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Abstract. The standard theoretical literature has shown that environmental sustainability and positive economic growth are not incompatible as long as environmental policies are optimal. However, in showing this result earlier studies have relied on strong assumptions that may appear to charge the dice in favor of such result. Here we show that once the role of the consumption composition effect is recognized, environmentally sustainable economic growth may exist even if some of the most questionable assumptions used by the canonical models are relaxed. In particular, we show that sustainable growth is possible even if environmental and man-made factors of production are complement rather than highly substitutable as has been invariably assumed by the literature and even if technological change is entirely pollution-augmenting.

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1. Introduction

This paper shows that environmentally sustainable economic growth is a likely outcome over the long run under much more general conditions than those often assumed by existing growth models. Our main contribution is the explicit consideration of more than one final goods, a feature that allows for the emergence of an output composition effect, an effect neglected by the standard growth literature which assumes a single final good. We show that the ability of consumers to substitute among clean and dirty final goods is a powerful and much neglected mechanism for sustainable growth. We show that consumer substitution, triggered by an optimal environmental tax, may induce sustainable growth even if man-made and environmental factors of production are complements rather than substitutes and even if we relax other strong assumptions often used by standard growth models.

The question of whether environmental degradation will eventually impose limits on economic growth has been a prominent theme in the literature (Stokey 1998; Copeland and Taylor, 2004; Arrow et al., 2010; Acemoglu et al. 2012). Using rather strong assumptions these studies have concluded that economic growth leads to policies and institutions which may make permanent economic growth compatible not only with a stable environment, but also eventually with an improving environment (López, 1994; Stokey, 1998; Acemoglu et al., 2012).

We argue that the existing formal growth models have reached this conclusion by imposing unnecessarily restrictive assumptions and omitting important adjustment mechanisms that have nonetheless been highlighted by the empirical literature. Most growth models assume one final good (e.g., Bovenberg and Smulder, 1995; Bovenberg and Mooij, 1997; Stokey, 1998; Bretschger and Smulders, 2007; Fullerton and Kim, 2008; Brock and Taylor, 2010; Acemoglu et al., 2012) which precludes the existence of an output composition effect, often considered important by empirical analyses (e.g., Grossman and Krueger, 1995; Antweiler et al., 2001; Cole and Elliot, 2003). In addition, standard growth models assume a greater-than-one elasticity of marginal utility of income, an assumption that has been criticized by prominent authors (e.g. Aghion and Howitt, 1997), and empirically questioned (Mulligan, 2002; Vissing-Jorgensen and Attanasio, 2003; Gruber, 2006; Layard et al., 2008). This assumption charges the dice in favor of sustainable growth because it imposes a form of satiation as it implies that the marginal utility of consumption falls very rapidly with consumption or income.

Most existing models assume production technologies that are Cobb-Douglas which impose a unitary elasticity of substitution between the man-made and environmental inputs while some more recent ones use CES specifications but assume highly elastic substitution. This assumption has been seriously challenged by environmentalists who claim that natural capital (the environment) and man-made capital are complements rather than substitutes (Daly 1992, 1994).[1] Moreover, empirical studies seem to support to some degree the claims by environmentalists as they have concluded that input substitution is indeed very limited; the estimated elasticity of substitution between clean and dirty inputs tends to be substantially less than one (see for example, Burniaux et al., 1991; Kemfert and Welsch, 2000; Van der Werf, 2007; Okagawa & Ban, 2008).[2]

By contrast, empirical studies of consumption substitution between clean and dirty consumer goods report much stronger substitution (see for example, Glaser and Thompson,

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[1] "The upshot of these considerations is that natural capital (natural resources) and manmade capital are complements rather than substitutes. The neoclassical assumption of near perfect substitutability between natural resources and manmade capital is a serious distortion of reality, the excuse of "analytical convenience" (Daly, 1992).

[2] See also Magnus (1979), and Field and Grebestein (1980) who found negative values of estimated Allen partial elasticity of substitution between manmade capital and energy. The negative sign of Allen elasticity signifies complementarity between man-made capital and energy

2000; Thompson and Glaser, 2001; Wier, Hansen and Smed, 2001; Lin et.al., 2008; Galarraga et. al., 2011). In fact, most studies report a large degree of substitution between environmentally mild consumer goods (such as organic products or high efficiency appliances) and conventional ones, reporting elasticity of substitution estimates well above 3. Thus, it appears that the scope for substitution between clean and dirty goods by consumers is much greater than the substitution potential among inputs by producers. This makes the neglect of the consumption composition effect by the standard single output models most unfortunate.

An exogenous technological change which is essentially factor augmenting for the clean input is also allowed in some studies (e.g. Stokey, 1998; Brock and Taylor, 2010). This assumption dramatically raises the likelihood of sustainability. An important exception is Acemoglu et al. (2012) which allows for endogenous factor-augmenting technological change in a model of constant elasticity of substitution technology between the clean and the dirty inputs although it retains the assumption of one final good. Acemoglu et al. (2012) claim that an optimal pollution tax, in combination with a (temporary) subsidy to R&D for the clean input sector sufficient to transform pollution-augmenting technical change into clean input-augmenting technical change, may cause sustainable economic growth as long as the production elasticity of substitution between the clean and the dirty inputs is greater than one.

Under the above-stated assumptions most existing growth studies have concluded that an optimal pollution tax is both necessary and sufficient for sustainable growth. However, the use of such restrictive and seemingly charged assumptions in favor of sustainable growth by standard models leaves the question of sustainable growth wide open. Are the canonical growth models really imposing rather than showing sustainable growth? We show that once the important consumer composition effect is considered, sustainable economic growth arises under much more general and arguably natural conditions than those commonly assumed by the growth literature. We focus on consumer flexibility as a key mechanism for sustainable growth under the simplest conditions and avoiding technical complications that could cloud the role of such flexibility. In particular, following much of the literature we focus on pollution flows omitting pollution stock effects (see for example, Gruver, 1976; Gradus and Smulders 1993; Copeland and Taylor, 1994; Andreoni and Levinson 2001; Copeland and Taylor, 2004; Levinson and Taylor, 2008) and consider only exogenous pollution-augmenting technological change. Consideration of pollution stock effects or endogenous rather than exogenous technological change is certainly important, but such considerations are unlikely to change the role of consumer substitution flexibility in sustainable growth.[3]

The present study develops a model with two final goods and generalizes other restrictive features of the existing growth models. The paper assumes the worse case scenario with respect to technical change by assuming pollution-augmenting technical progress; it does not impose a greater-than-unity elasticity of marginal utility of income and allows for a less-than-one elasticity of substitution between the clean and dirty inputs in production. The paper shows plausible conditions under which an optimal pollution tax may indeed be necessary and sufficient for sustainable development.

2. Framework of the analysis

The economy produces two goods: a clean and a dirty one. The dirty good sector includes traditional manufacturing industries and primary industries that generate air and/or

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[3] Alternatively, one may consider the analysis to be directly relevant to the case of pollutants that dissipate quickly such as local ozone, nitrogen dioxide, air particles and many others. Similarly, the assumption of technological change being completely pollution-augmenting can be regarded as the result of a market allocation of R&D without subsidies as in Acemoglou et al. (2012).

water pollution as a byproduct of their production processes, while the clean good sector includes services and other goods that generate little or no pollution.

Let k denote the total man-made composite input available at time t in the economy. This composite input includes human capital, the stock of knowledge, as well as other more tangible forms of capital. Henceforth, we refer to k as "capital", which is momentarily distributed between the clean industry and dirty industry. Let k_d denote the amount of capital employed in the dirty industry. The flow of pollution from the dirty sector is represented by x. Following Cropper and Oates (1992), López (1994), and Copeland and Taylor (1994, 2004), we consider pollution as a factor of production directly . The output of the dirty sector is then,

(1) $$y_d = A_D F(k_d, bx).$$

Where the parameter A_D denotes total factor productivity and b is a pollution-augmenting technological factor which is assumed to change over time, so that $\dot{b}/b \equiv \zeta$. The dirty sector produces only a final consumer good. The function F is characterized by a constant elasticity of substitution (CES) functional form,

$$F(k_d, bx) = \left[\alpha k_d^{-\frac{1-\omega}{\omega}} + (1-\alpha)(bx)^{-\frac{1-\omega}{\omega}}\right]^{-\frac{\omega}{1-\omega}},$$

where ω is the elasticity of substitution between capital and pollution and α is a fixed distribution coefficient.

The output of the clean good sector is assumed to depend only on the capital input and is governed by the linear production technology,

(2) $$y_c = A_C(k - k_d),$$

where the parameter A_c is the return to capital in the clean sector and k is the total stock of capital in the economy at a point in time. Unlike the dirty sector which produces only final goods, the clean sector produces a final consumer good as well as new capital goods (or investment goods).

If we normalize the price of the clean good to unity ($p_c = 1$), the economy's budget constraint can be written as,

$$\dot{k} = A_C(k - k_d) + pA_D F(k_d, x) - c - \delta k, \tag{3}$$

where $p \equiv p_d / p_c$ is the relative price of the dirty good, $c \equiv c_c + pc_d$ is the total consumption expenditure expressed in units of the clean good, δ is the rate of capital depreciation, and $\dot{k} \equiv dk/dt$ is the net capital accumulation. The sum of the first two terms on the right-hand side of equation (3) represents the income of the economy expressed in units of clean goods. The gross capital accumulation, $\dot{k} + \delta k$, is equal to net savings (income less consumption), also expressed in units of the clean goods.[4]

We assume that the consumer's indirect utility function is as follows:

$$u = \frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a},$$

where c denotes the total consumption expenditure, $e(1,p)$ is the unit (dual) expenditure function or cost-of-living index, and $a > 0$ is a parameter corresponding to the elasticity of marginal utility of income or consumption (*EMU*). If $a < 1$, we adopt a positive utility scale such that $0 < u < \infty$, while we scale the utility index to $-\infty < u < 0$ when $a > 1$. The indirect

[4] We assume that the investment in capital is irreversible. Once the economy builds capital, it cannot be transformed back into consumption goods and thus capital can be reduced through time only by allowing it to depreciate.

utility function is thus assumed to be increasing and strictly concave in the real consumption level, $c/e(1,p)$.

We assume that the consumer's underlying preferences are described by a CES utility function so that the unit expenditure function is,

$$e(1,p) = \left[\gamma_c + \gamma_d p^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

where σ is the consumption elasticity of substitution between a dirty good and clean good, and $\gamma_c > 0$ and $\gamma_d > 0$ are fixed parameters. The indirect utility function defined above presumes homothetic preferences. Consumer demand for the clean good c_c and dirty good c_d can be retrieved from the indirect utility function using Roy's identity. The optimal level of c is determined by the inter-temporal optimization, as detailed below.

Let $v(x)$ denote the environmental damage function, which is assumed to be increasing and convex in the level of pollution, x. We assume that the environmental damage function is $v(x) = \frac{x^{1+\eta}}{1+\eta}$, where η>0 denotes the elasticity of marginal damage of pollution and is assumed to be a fixed parameter. Then the consumer's instantaneous welfare is

$$U \equiv \frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a} - \frac{x^{1+\eta}}{1+\eta}.$$

Assuming a fixed pure time discount rate (ρ) and that the government regulates pollution emissions in an optimal way, the competitive economy behaves "as if" it maximizes the present discounted value of the utility function,

$$\int_0^\infty \left\{\frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a} - \frac{x^{1+\eta}}{1+\eta}\right\}\exp(-\rho t)dt,$$

subject to the budget constraint (equation (3)), and the initial condition $k = k_0$. It does so by choosing the optimal levels of c and x at each point in time. The government imposes a pollution tax in a socially optimal way and reimburses the tax revenue in a lump-sum way to the consumer. The above optimization implies the following current value Hamiltonian function,

$$H = \frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a} - \frac{x^{1+\eta}}{1+\eta} + \lambda\left[A_C(k-k_d) + pA_D F(k_d, bx) - c - \delta k\right],$$

where λ is the shadow price of capital (also equal to the marginal utility of consumption).

Before proceeding with the analysis, we need to define what we mean by "sustainable growth." Hence, the following definition: *We say that sustainable growth is possible if, at some point along the growth process, the economy is able to continue growing indefinitely while pollution emissions stop increasing or even decline.*

This definition implies that there are at least two cases under which sustainable economic growth occurs: one is the case of monotonic reductions of pollution emissions along the full growth process or, alternatively, where this relationship is not monotone, so that pollution may follow any pattern until it stops increasing or starts falling indefinitely over time. In mathematical terms, sustainability requires that there exists a finite time $T \geq 0$ such that at any time $t > T$, $\hat{x} \leq 0$. If the pollution level changes in a continuously differentiable manner over time, the sustainability condition is satisfied if $\lim_{t\to\infty} \hat{x} < 0$.[5]

3. Optimality conditions

The first-order necessary conditions for maximization of the Hamiltonian function, imply that the marginal utility of consumption must be equal to the shadow price of capital, λ,

[5] A similar notion has been adopted by several authors including Acemoglu et al. (2012) and Stokey (1998).

(4) $$e(1 p^{a})^{-1} c^{-a} = \lambda$$

The shadow price of capital decreases if the capital accumulates over time,

(5) $$\frac{\dot{\lambda}}{\lambda} = -\left[A_C - \rho - \delta\right] \equiv -M.$$

Additional conditions for optimality are that the marginal value product of capital should be equal across the two sectors and that firms equalize the marginal value product of pollution to the optimal pollution tax. Thus assuming an interior solution we have,

(6) $$pA_D \frac{\partial F(k_d, bx)}{\partial k_d} - A_C = 0$$

(7) $$-v'(x) + \lambda p A_D \frac{\partial F(k_d, bx)}{\partial x} = 0.$$

Equation (6) indicates that in equilibrium the marginal value product of capital should be equalized across the two sectors. Equation (7) says that the optimal pollution tax should be equal to the marginal rate of substitution between pollution and consumption, $\tau^* \equiv v'(x)/\lambda$, which, in turn is equalized to the marginal value product of pollution. Also, the savings should be equal to the net investment at each moment of time so that we have equation (3) as an additional first order condition. Finally, we have the standard transversality condition, $\lim_{t\to\infty} \lambda k(t) e^{-\rho t} = 0$.

On consumption and factor shares

The budget share of dirty final good in the consumption expenditure for the CES utility function is $s(p) = \dfrac{\gamma_d}{\gamma_c p^{\sigma-1} + \gamma_d}$ and the factor share of the clean input in the cost of production of the dirty good is $S_k(k_d/bx) = \alpha\left[(1-\alpha)\left(\dfrac{k_d}{bx}\right)^{\frac{1-\omega}{\omega}} + \alpha\right]^{-1}$. Of course the share of the dirty input in the cost of production of the dirty final good is $(1-S_k)$. Then we have the following remark,

Remark 1: *The share* $s(p)$ *is an increasing (decreasing) function of* p *if* $\sigma < 1$ $(\sigma > 1)$. *The share* $S_k(k_d / bx)$ *is increasing (decreasing) in* k_d / bx *if* $\omega > 1(\omega < 1)$.

The following lemma states the conditions under which pollution-augmenting technological change increases the marginal value product of pollution and hence induces the profit maximizing firm to increase the pollution for a given level of pollution tax.

Lemma 1: *An increase in the rate of pollution-augmenting technical progress increases the marginal product of pollution if and only if* $S_k < \omega$.

Proof: *See the Appendix.*

The condition for Lemma 1 is certainly satisfied when $\omega > 1$. If the lemma 1 holds, we can say that the technical progress is complementary with pollution. The firm facing the complementary technical progress would find it more costly to substitute pollution by capital than in the absence of technical progress.

4. Assumptions

Before analyzing the dynamic properties of the model we make the following assumptions,

Assumption 1: The clean sector of the economy is sufficiently productive so that the marginal return to capital (A_c) is higher than the marginal opportunity cost of capital ($\rho + \delta$); hence, $M \equiv A_C - \rho - \delta > 0$.

Assumption 2: Technical change is exclusively pollution-augmenting occurring at an exogenous rate ζ >0. However, the rate of pollution-augmenting technical change is bounded from above as follows: $\zeta \leq min\{M, M / a\}$.

Assumption 3 (to be relaxed later): The elasticity of marginal utility of consumption, a, is greater than one.

Assumption 1 is a necessary condition for the economy to be able to grow over time. Assumption 2 implies that all technical change is concentrated on augmenting the dirty input while the clean input does not augment its productive capacity.[6] Thus, we consider a context where technical progress is most unfavorable to sustainable growth. However, Assumption 2 also places a limit on the speed of pollution-augmenting technical progress. As we show below this limit is necessary to preserve the existence of the technique and composition effects. If this assumption is not satisfied the clean-to-effective pollution ratio (k_d / bx) and relative price of the dirty final good would continuously decrease through time. This would imply that sustainable development almost by definition would not be feasible because there would not be any counterbalance to the pollution-increasing scale effect associated with economic growth. Assumption 3, which is later relaxed, is made mostly for comparing our findings with the standard literature, which has invariably made it.

5. Dynamic market equilibrium conditions

The logarithmic differentiation of (4) with respect to time yields $a\hat{c} + (1-a)\hat{e}(1,p) = -\hat{\lambda}$ where $\hat{c} \equiv \dot{c}/c$, $\hat{e} \equiv \dot{e}/e$ and $\hat{\lambda} = \dot{\lambda}/\lambda$ denote the proportional rate of change of each variable. Also, from (4) and (5), and using Shephard's lemma, it follows that $\hat{e}(1,p) = \frac{pe_2}{e}\hat{p} = s(p)\hat{p}$.

In the Appendix we show that the rate of growth of the consumer demand for the dirty

[6] In the context of the Acemoglu et al. (2012) model of endogenous technical change this assumption may be interpreted to imply a corner solution, where all the innovative resources (scientists, for example) are invested in augmenting the dirty input and no innovative resources are spent in augmenting the clean input, which corresponds to the market solution without research subsidies.

good is,

(8) $$\hat{c}_d = \frac{1}{a}M - \left[\frac{s(p)}{a} + (1-s(p))\sigma\right]\hat{p}.$$

Also, the rate of growth of production of the dirty good is

(9) $$\hat{y}_d = \hat{F}(k_d, bx) = S_k \widehat{\left(\frac{k_d}{bx}\right)} + \widehat{(bx)}.$$

Since the dirty good is used for consumption only, market equilibrium requires that $y_d = c_d$ at all points in time. Furthermore, once the dirty good market is cleared, the market for the clean good is automatically cleared since the current savings are equal to the current investment as stipulated in equation (3). Thus, the price of the dirty good must adjust endogenously over time to allow for such equilibrium to persist. Along the equilibrium path, the growth rate of production and demand for the dirty good must be equal, so that $\hat{y}_d = \hat{c}_d$; using (8) and (9), we obtain,

(10) $$z\hat{p} + S_k \widehat{\left(\frac{k_d}{bx}\right)} + \hat{x} = \frac{M}{a} - \zeta \quad ,$$

where $z \equiv \frac{s(p)}{a} + (1-s(p))\sigma > 0$.

From (6), we also have that $\hat{p} + \hat{F}_1(k_d, bx) = 0$, which given the CES production function implies,

(11) $$\hat{p} - \frac{1}{\omega}(1-S_K)\widehat{\left(\frac{k_d}{bx}\right)} = 0\,.$$

Finally, in the Appendix we show that using equation (7) the following expression follows,

(12) $$\hat{p} + \frac{1}{\omega}S_K \widehat{\left(\frac{k_d}{bx}\right)} - \eta\hat{x} = M - \zeta.$$

It states that the rate of increase of the private marginal revenue of the dirty input, $\hat{p}+\frac{1}{\omega}S_K\widehat{\left(\frac{k_d}{bx}\right)}+\zeta$, is equal to the rate of increase of the input price, which in turn equals rate of increase of the pollution tax, $\eta\hat{x}+M$.

5.1 Solution of the system dynamics

In the Appendix we show that the dynamical system of equations (10), (11), and (12) solves for the equilibrium growth rates of $\hat{p}$, $\widehat{\left(\frac{k_d}{x}\right)}$ and $\hat{x}$ as follows:

(13) $$\hat{p}=\frac{1}{|W|}\frac{(1-S_k)}{\omega}\left(\left(M/a-\zeta\right)\eta+M-\zeta\right)>0$$

(14) $$\widehat{\left(\frac{k_d}{bx}\right)}=\frac{1}{|W|}\left[\left(M/a-\zeta\right)\eta+M-\zeta\right]>0$$

(15) $$\hat{x}=\frac{1}{|W|\omega}\left\{M/a-\zeta-\omega S_k[M-\zeta]-z(1-S_k)[M-\zeta]\right\}$$

where $|W|\equiv\frac{1}{\omega}\left[(1-S_k)(1+z\eta)+S_k\right]+\eta S_k>0$.

From equation (15) it follows that the dynamics of pollution can be decomposed into four partial effects:

(15’) $$\hat{x}=\frac{1}{\omega|W|}[\varepsilon_y+\varepsilon_t+\varepsilon_s+\varepsilon_c]$$

where $\varepsilon_y\equiv M/a>0$ is the *pure scale effect*; $\varepsilon_t\equiv-\zeta<0$ is the *pure technological change effect*; $\varepsilon_s\equiv-\omega S_k[M-\zeta]<0$ is *technique effect*; $\varepsilon_c\equiv-z(1-S_k)[M-\zeta]<0$ is the *output composition effect.*

The pure scale and technological change effects are autonomous, caused by the two primary sources of economic growth, capital accumulation and technological change, respectively, while the technique and output composition effects are dependents on the two autonomous factors. The pure scale effect, ceteris paribus, increases pollution as it shows the effect of factor expansion while the pure technological change effect reduces pollution because it reflects the fact that the effective dirty input rises over time without increasing pollution. But scale and technological change also cause two indirect effects, the technique or input substitution effect and the output composition effect. Scale induces an increase of the pollution tax due to the fact that the marginal utility of consumption, λ, falls as M>0, which in turn triggers the technique or input substitution effect, which has a pollution reducing effect. Scale also triggers an output composition effect because the pollution tax increase associated with the scale effect raises the cost of production of the dirty good. This, in turn, increases the relative price of the dirty good inducing consumers to substitute consumption of dirty goods with clean goods. However, pollution-augmenting technological change weakens both the technique and composition effects. The increase of the productivity of pollution due to technological change counters the effect of the increased pollution tax because the relative price of effective pollution increases less making the incentives to substitute pollution with clean inputs weaker. Similarly, the increased productivity associated with technological change attenuates the cost increase of the dirty good caused by the pollution tax. This, in turn, reduces the price increase of the dirty good and hence lowers the consumers' incentives to substitute dirty goods with clean ones.

Both the technique and composition effects are triggered by the fact that along the growth path the optimal pollution tax, $\tau \equiv v'(x)/\lambda$, is increasing. The reason is that since by Assumption 1 ($M > 0$) the shadow value of capital, λ, is constantly falling. If pollution increases then this effect is exacerbated because in this case $v'(x)$ also rises. If pollution is falling then $v'(x)$ would be decreasing, only partially mitigating the effect of the decline of λ. As τ increases the cost of the dirty input vis-à-vis the price of the clean input, k_d, may increase as long as the pollution-augmenting technical change effect is not too large (the price of k_d remains constant being equal to A_c). In fact, Assumption 2 assures that the technological change effect is not too strong and does not dominate the tax effect, so that the price of pollution relative to the price of the clean input does increase over time. The assumption 2 also assures that the cost of production of the dirty final good increases over time, which in turn triggers the consumption substitution or composition effect in favor of the clean final good.

An important issue is whether the dynamic path described by equations (13) to (15) yields a positive rate of consumption growth. Lemma 2 below shows that this is indeed the case.

Lemma 2: *(i) The growth rate of real consumption expenditure is,*

$$\widehat{\left(\frac{c}{e}\right)} = \frac{1}{a}\left[M - s(p)\hat{p}\right].$$

(ii) The rate of growth of real consumption remains positive along the equilibrium dynamic path. (iii) If either input substitution or consumption substitution is elastic (if $\omega > 1$ or $\sigma > 1$) the rate of growth of real consumption converges from below towards a rate M/a. (iv) If $\omega < 1$ and $\sigma < 1$ then the rate of growth of real consumption converges from above towards $\frac{1+\eta}{a+\eta}\zeta$.

Proof: *See Appendix.*

Lemma 2 shows that the dynamic equilibrium path described by equations (13) to (15) is associated with a positive rate of growth of real consumption. But the economy's growth rate is below its potential as a consequence of the fact that the optimal pollution tax forces the price of the dirty goods to continuously increase over time. This, in turn, increases the cost of living for consumers implying that economic growth must be partially sacrificed. However, as shown in Remark 1, if $\sigma > 1$ the share of the dirty good in the consumption bundle declines and if $\omega > 1$ the share of the clean input in production increases. In either of these cases the sacrifice of the growth rate vis-à-vis its potential level, becomes progressively smaller as time goes on. That is, the growth rate of the economy approaches in the long run its maximum potential rate, which is equal to M / a.

If $\sigma > 1$ or $\omega > 1$ we have that the convergence rate of growth of the economy is not affected by the rate of technological change. The reason for this is that in this case the consumer budget share of pollution and the share of pollution in the cost of production approach zero.[7] That is pollution-augmenting technical change becomes irrelevant for economic growth over the very long run, and hence the convergence growth rate is not affected by technical change; only the net productivity of capital (as well as the size of EMU) is relevant. For finite time this means that in the elastic case the economy's growth rate is increasing, becoming less and less dependent on the rate of technological change (and more dependent on the rate of capital accumulation) as the share of pollution in production and/or the consumer budget share of the dirty good fall over time.

Also, from Remark 1 it follows that if $\omega < 1$ and $\sigma < 1$ then the share of the dirty input (pollution) in the cost of production increases over time and the share of the dirty good in the consumer budget increase over time, converging towards 1. Thus, the (pollution-augmenting) technological change becomes the key determinant of the convergence rate of economic growth.

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[7] This is true if $\sigma > 1$ but $\omega < 1$ because in this case the consumption share of the dirty good approaches zero and hence the participation of the dirty good in the economy becomes negligible in the very long run. Also, if $\sigma < 1$ but $\omega > 1$ the share of pollution in the dirty good approaches zero, meaning that in the very long run the participation of pollution as an input becomes negligible.

On the other hand, since the share of the clean good approaches zero the capacity of the economy to expand it becomes progressively irrelevant for economic growth. This means that in the inelastic case the economy's growth rate declines and becomes more and more dependent on the rate of technological change and less dependent on the rate of capital accumulation as the shares of the dirty input and dirty final output increase over time. Moreover, by Assumption 2 it follows that the growth rate of the economy converges to a lower level than in the elastic case.

This analysis highlights the importance of the producer and consumer flexibility for economic growth. Economies characterized by elastic producer and/or consumer choices tend to grow faster converging towards higher secular growth rates than economies exhibiting inelastic producer and consumer choices.

4.2.2 Conditions for Sustainable Growth

We now investigate the conditions for sustainable growth through the output composition effect and input composition or technique effect.

The output composition effect ($\sigma \geq 1; \omega < 1$ ***)***

The composition effect works when consumers substitute dirty goods with clean goods in the face of rising relative price of the dirty good. Given assumptions 1 and 2, $M\left(\frac{1+(\eta / a)}{1+\eta}\right)-\zeta > 0$ for any $\eta \geq 0$. Then, according to the equation (13), the price of the dirty good rises throughout the dynamic path.

First we consider the case where the consumption elasticity of substitution is strictly greater than 1 but the production elasticity of substitution is less than 1. In this case the input composition effect becomes important and the feasibility of sustainable growth relies exclusively on the consumer flexibility. Since $\left(\frac{\hat{k}_d}{bx}\right)$ increases over time, the factor share of the clean input in the output value of the dirty final good, S_k, converges to zero (and concomitantly, the share of the dirty input converges towards 1). Sustainable growth is then feasible only if production of the dirty good start declining at some point along the dynamic path. The fact that the price of the final dirty good continuously increases over time means that consumers would substitute dirty goods with clean ones. However, this substitution effect must be sufficiently powerful to offset the scale effect arising from the fact that real income is increasing. Therefore, the strength of the substitution effect is crucial; if the elasticity of substitution in consumption is greater than one ($\sigma > 1$), the budget share of the dirty good, $s(p)$, decreases toward zero over time. This implies that at some point along the dynamic growth path the demand for the dirty good (and hence its production) may start shrinking, eventually allowing pollution to fall. On the other hand, if $\sigma < 1$ the substitution effect will be too weak and the income effect would dominate, thus precluding sustainable growth altogether.

Thus, assuming that $\sigma > 1$ and $\omega < 1$ the limit to equation (15) is,

(16) $$\lim_{t\to\infty} \hat{x} = \frac{1}{\omega} \frac{M\left(\frac{1}{a} - \sigma\right) - \zeta(1-\sigma)}{(1+\sigma\eta)}.$$

From (16) it follows that,

$$\lim_{t\to\infty} \hat{x} < 0 \text{ if and only if } \zeta < \frac{M}{a}\left(\frac{a\sigma - 1}{\sigma - 1}\right).$$

Assumptions 1 and 2 imply that $\zeta < M/a$. Hence, sustainable growth is feasible if the term in brackets in the previous expression is greater or equal to one. That is, as long as $\sigma > 1$ we have sustainable development.

Now consider the case of Cobb-Douglas preferences ($\sigma = 1$). Then under the Assumption 2, the equation (15) implies that $\lim_{t\to\infty} \hat{x} < 0$ if and only if $M\left(\frac{1}{a} - z\right) - \xi(1-z) < 0$.

Since $a > 1$, and $\sigma = 1$, we have $\frac{1}{a} < z \leq 1$ and $\lim_{t\to\infty} \hat{x} < 0$, and the condition for sustainability is satisfied.

The following proposition summarizes the previous results,

Lemma 3 (on the role of the composition effect). *An inelastic production elasticity of substitution does not preclude sustainable economic growth as long as the consumer elasticity of substitution is greater or equal to one.*

Lemma 3 underlines the importance of the composition effect to circumvent the case of a highly inelastic production technology. All previous analyses assuming a single final good and, hence ignoring the output composition effect, have concluded that a flexible production technology ($\omega \geq 1$) is a necessary condition to allow for sustainable development. Lemma 3 shows that this is not true as long the consumer preferences are sufficiently flexible ($\sigma \geq 1$) in which case the composition effect dominates the scale effect. All that is needed in this case is an optimal pollution tax that forces the relative price of the dirty final good to increase over time. Remarkably, sustainable growth under an optimal pollution tax with $\sigma \geq 1$ occurs even if the production function of the dirty good is Leontief ($\omega = 0$), that is, even if clean and dirty inputs are complements.

The Input substitution or technique effect ($\omega \geq 1; \sigma < 1$***)***

We first consider the case where the technical elasticity of substitution between the two inputs is strictly greater than one while the consumption elasticity of substitution is less than one;

in this case the cost share of the clean input approaches one while the share of the dirty good in the consumer budget also approaches 1. The feasibility of sustainable growth depends solely on technique effect. From the equation (15) we have,

(17) $$\lim_{t\to\infty}\hat{x}=\frac{M\left(\frac{1}{a\omega}-1\right)-\zeta\left(\frac{1}{\omega}-1\right)}{\frac{1}{\omega}+\eta}.$$

The first term of the numerator represents the technique effect that results from a change in the relative factor costs of production. The optimal pollution tax makes the pollution input expensive to use, and if the technical elasticity of substitution between the clean and the dirty input is greater than one the pollution input is gradually substituted with capital. However, due to factor augmenting technological progress, we know that if the productivity of the pollution input grows over time, it increases the marginal product of pollution and makes it costly to substitute the pollution input with capital. The second term of the numerator denotes the productivity effect of pollution. Only if the technique or substitution effect outweighs the technical change effect, sustainable growth becomes possible. Since $\frac{(a\omega-1)}{a(\omega-1)}M>\zeta$ it is straightforward to find that if $\omega>1$ then $\lim_{t\to\infty}\hat{x}<0$.

Finally, if the production function is Cobb-Douglas, $\omega=1$, the cost share of capital remains constant and equals α. From the equation (13), the price of the dirty final good always increases over time and the budget share of the dirty good converges to zero.

(18) $$\lim_{t\to\infty}\hat{x}=\frac{1}{1+\eta}\left(\frac{1}{a}-1\right)M<0.$$

Thus we have Lemma 4,

Lemma 4 (on the technique or input composition effect). *An inelastic consumption elasticity of substitution does not preclude sustainable economic growth as long as the elasticity of substitution between the clean and dirty inputs is greater or equal to one.*

Importantly, a sufficient condition for sustainable growth is that $\omega \geq 1$ even if technological change is entirely pollution-augmenting. In our model (unlike the model in Acemoglu et al., 2012, for example) capital (the clean input) is expanding in a growing economy. Hence, even if technological change is only pollution-augmenting, as we assume, the capital-to-effective pollution ratio (k_d / bx) may increase without requiring a very rapid increase of the pollution tax. That is, the technique effect does not rely exclusively on the pollution tax; it is reinforced by the capital growth effect. Thus, if the elasticity of substitution between capital and pollution is greater or equal to one, the substitution effect may dominate the expansion effect within the dirty sector and pollution may decrease at some point along the growth path.

By contrast, in the Acemoglu et.al model growth relies exclusively on technological change; in their analysis inputs can grow only by increasing their respective effective levels through input-augmenting technological change. Thus, the substitution effect arises if technological change in the clean input sector is faster than in the dirty input sector; this requires a subsidy to R&D to the clean input sector so that the effective clean input/effective pollution ratio grows. Otherwise, the required pollution tax must increase at a very rapid rate. In our model the stock of capital is continuously increasing, meaning that the capital input in the dirty sector may increase even if the pollution tax increase at a slower rate.

Combining Lemmas 3 and 4 we obtain the following proposition,

Proposition 1: S*ustainable growth is feasible if either the elasticity of substitution between clean and dirty final goods is greater than or equal to one ($\sigma \geq 1$), or the technical elasticity of* *substitution between the clean and the dirty inputs is greater than or equal to one(* $\omega \geq 1$*).*

Proposition 1 states that even if the technical progress is biased toward the dirty input in a pollution-augmenting fashion and the technical elasticity of the substitution between the clean and dirty inputs is less than unity, an optimal pollution tax is sufficient to induce environmental sustainability while still allowing for a positive rate of economic growth as long as the consumer preferences are flexible enough. Similarly, Proposition 1 shows that even if consumer preferences exhibit little flexibility ($\sigma < 1$) an optimal pollution tax is sufficient to allow for sustainable growth if input production substitution is elastic ($\omega > 1$). Thus, as long as the technical progress rate in the dirty sector is reasonably bounded, the optimal pollution tax alone can lead the sustainable growth, and the subsidy is not necessary even when the technical progress is concentrated in the dirty sector only.

4.3 Conditions for Sustainable Growth: Extension

We now consider the case where the elasticity of marginal utility, a, is less than one. This generalization is important especially in view of the fact that the standard growth models assume that $a > 1$, a highly restrictive imposition upon the structure of preferences. The main consequence of removing the standard assumption and allowing $a < 1$ is that in this case the rate of economic growth is more rapid than in the case where $a > 1$ (due to the fact that when $a < 1$ the marginal utility of consumption decreases at a slower rate). That is, the scale effect is more powerful and hence, ceteris paribus, pollution will tend to grow faster as the economy grows. This means that sustainable growth will require stronger substitution effects.

Using equation (16), the following corollaries emerge,

Corollary 1: *Suppose that the elasticity of marginal utility of consumption is less than one. If* *the consumption elasticity of substitution is sufficiently large so that* $\sigma > \frac{1}{a}$ *, and the rate of technical progress,* ζ *, is bounded by* $\left(\frac{\sigma - 1/a}{\sigma - 1}\right)M$ *then sustainable growth is feasible regardless of the size of the input elasticity of substitution.*

Proof: *See Appendix.*

In a similar vein, we show the following corollary.

Corollary 2: *Suppose that the elasticity of marginal utility of consumption is less than one. If the technical elasticity of substitution is sufficiently large so that* $\omega > \frac{1}{a}$*, and the technical progress rate,* ζ *, is bounded by* $\left(\frac{\omega - 1/a}{\omega - 1}\right)M$ *, then sustainable growth is possible regardless of the consumption elasticity of substitution.*

Proof: *See Appendix.*

We first note that in both corollaries, the upper bounds on the pollution-augmenting technical progress rate are now lower than in the case when $a > 1$. In addition, the minimum size of either the consumption or production elasticity of substitution is now higher than in the case when $a > 1$. However, it is important to emphasize that if $a < 1$ sustainable growth is still feasible even in the case where production flexibility is very low, including the case of a Leontief technology.

5. Conclusion

Most of the existing literature on sustainable growth is based on one final good model where the production technology is assumed to exhibit elastic substitution between clean and dirty inputs. In addition, previous growth models have assumed that the elasticity of marginal utility of consumption is greater than 1 despite that this assumption has raised substantial conceptual concerns and that is inconsistent with the findings of at least some empirical studies. Also, the assumption of an elastic substitution between dirty and clean inputs contradicts the available empirical evidence that shows that production substitution is quite weak. In fact, most empirical studies that measure the elasticity of substitution between clean and dirty inputs obtain values well below one.

This paper has shown that, if consumer preferences between the clean and dirty goods are flexible enough, then the optimal pollution tax alone can achieve sustainable development even if the production technology is inflexible as empirical studies have shown. It holds even if technological change is 100% pollution-augmenting and even if the elasticity of marginal utility of consumption is less than one. In contrast with the assumption of high producer flexibility made by standard growth models, our assumption of consumer flexibility appears to be better supported by empirical studies.

The paper did not examine the short term behavior of the pollution-income relationship and focuses mostly on the long-run properties of economic growth under environmental regulation. The dynamic properties of pollution-income relationship in a growing economy remain an interesting area of research.

Appendix

Proof of Lemma 1:

The marginal product of pollution is defined as

(A1) $$\frac{\partial F(k_d, bx)}{\partial x} = bF_2$$

Therefore using the CES production function, we have

(A2) $$\frac{\partial (bF_2)}{\partial b} = F_2\left(1 + bx\frac{F_{22}}{F_2}\right) = F_2\left(1 - \frac{S_k}{\omega}\right)$$

Derivation of equation (8) :

We use Roy's identity to derive the demand for the dirty good from the indirect utility function as follows.

(A1) $$c_d = \frac{c}{e(1,p)} e_2(1,p)$$

Logarithmic time differentiation yields,

(A2) $$\hat{c}_d = \hat{c} + \hat{e}_2(1,p) - \hat{e}(1,p) .$$

Using Shephard's lemma we know that $\hat{e}(1,p) = \frac{pe_2}{e}\hat{p} = s(p)\hat{p}$

On the other hand, we have,

(A3) $\hat{e}_2 = \frac{d \log e_2}{dt} = \frac{e_{22}}{e_2} \frac{dp}{dt}$

where $\frac{e_{22}}{e_2} = \frac{d \log e_2}{dp}$ and $e_2 = \left(\gamma_c + \gamma_d p^{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}} \gamma_d p^{-\sigma}$.

Since $\log e_2 = \left(\frac{\sigma}{1-\sigma}\right) \log\left(\gamma_c + \gamma_d p^{1-\sigma}\right) + \log \gamma_d - \sigma \log p$

we have

(A4) $\frac{d \log e_2}{dp} = \left(\frac{\sigma}{1-\sigma}\right) \frac{\gamma_d (1-\sigma) p^{-\sigma}}{\gamma_c + \gamma_d p^{1-\sigma}} - \frac{\sigma}{p} = \frac{\sigma}{p}\left(s(p)-1\right)$

Hence,

(A5) $\hat{e}_2 = \frac{\sigma}{p}\left(s(p)-1\right) \frac{dp}{dt} = \sigma(s(p)-1)\hat{p}$

Therefore, from (A1), and equations (4) and (5), we arrive at equation (8) as follows;

(A6) $\hat{c}_d = \left(\frac{1-a}{a}\right)\left[M - s(p)\hat{p}\right] + \sigma(s(p)-1)\hat{p} - s(p)\hat{p}$

$$= \frac{1}{a} M - \left[\frac{s(p)}{a} + (1-s(p))\sigma\right]\hat{p}$$

<u>Derivation of equation (12):</u>

We first note that

(A7) $\frac{\partial F(k_d, bx)}{\partial x} = bF_2(k_d, bx)$

where,

(A7) $F_2(k_d, bx) = -\frac{\omega}{1-\omega}\left[\alpha k_d^{-\frac{1-\omega}{\omega}} + (1-\alpha)bx^{-\frac{1-\omega}{\omega}}\right]^{-\frac{1}{1-\omega}}(1-\alpha)\left(-\frac{1-\omega}{\omega}\right)(bx)^{-\frac{1}{\omega}}$

Then $LogF_2(k_d, bx) = -\frac{1}{1-\omega}\log\left[(bx)^{-\frac{1-\omega}{\omega}}\left\{(1-\alpha)+\alpha\left(\frac{k_d}{bx}\right)^{-\frac{1-\omega}{\omega}}\right\}\right] + \log(1-\alpha) - \frac{1}{\omega}\log(bx)$

$$= -\frac{1}{1-\omega}\log(bx)^{-\frac{1-\omega}{\omega}} - \frac{1}{1-\omega}\log\left[(1-\alpha)+\alpha\left(\frac{k_d}{bx}\right)^{-\frac{1-\omega}{\omega}}\right] + \log(1-\alpha) - \frac{1}{\omega}\log(bx)$$

$$= -\frac{1}{1-\omega}\log\left[(1-\alpha)+\alpha\left(\frac{k_d}{bx}\right)^{-\frac{1-\omega}{\omega}}\right] + \log(1-\alpha)$$

Therefore,

$$\frac{dLogF_2(k_d, bx)}{dt} = -\frac{1}{1-\omega}\frac{-\frac{1-\omega}{\omega}\alpha\left(\frac{k_d}{bx}\right)^{-\frac{1}{\omega}}\left(\frac{k_d}{bx}\right)\left(\frac{\hat{k_d}}{bx}\right)}{(1-\alpha)+\alpha\left(\frac{k_d}{bx}\right)}$$

$$= \frac{1}{\omega}\frac{\alpha\left(\frac{k_d}{bx}\right)^{-\frac{\omega-1}{\omega}}\left(\frac{\hat{k_d}}{bx}\right)}{(1-\alpha)+\alpha\left(\frac{k_d}{bx}\right)^{-\frac{\omega-1}{\omega}}} = \frac{1}{\omega}\frac{\alpha\left(\frac{\hat{k_d}}{bx}\right)}{\left(\frac{k_d}{bx}\right)^{-\frac{\omega-1}{\omega}}\left[(1-\alpha)+\alpha\left(\frac{k_d}{bx}\right)^{-\frac{\omega-1}{\omega}}\right]}$$, which implies that

(A8) $\left(F_2(\hat{k_d}, bx)\right) = \frac{\alpha}{\omega}\frac{\left(\frac{\hat{k_d}}{bx}\right)}{\left[(1-\alpha)\left(\frac{k_d}{bx}\right)^{-\frac{\omega-1}{\omega}} + \alpha\right]} = \frac{S_k}{\omega}\left(\frac{\hat{k_d}}{bx}\right)$

Then from (7) we have

$$\text{(A9)}\quad \eta\hat{x} - \hat{\lambda} = \hat{p} + \hat{b} + \frac{S_k}{\omega}\left(\frac{\hat{k_d}}{bx}\right)$$

Rearranging the above equation and using $\hat{b} \equiv \zeta$, we arrive at

$$\text{(A10)}\quad \hat{p} + \frac{S_k}{\omega}\left(\frac{\hat{k_d}}{x}\right) - \eta\hat{x} = M + \left(\frac{S_k}{\omega} - 1\right)\zeta$$

Derivation of equations (13), (14) and (15)

The system of equations (10), (11) and (12) in matrix form is,

$$\begin{bmatrix} z & S_k & 1 \\ 1 & -\frac{1}{\omega}(1-S_k) & 0 \\ 1 & \frac{1}{\omega}S_k & -\eta \end{bmatrix} \begin{bmatrix} \hat{p} \\ \left(\frac{\hat{k}_d}{bx}\right) \\ \hat{x} \end{bmatrix} = \begin{bmatrix} \frac{M}{a} - \zeta \\ 0 \\ M - \zeta \end{bmatrix}$$

Using Cramer's rule and noting that the determinant

$$|W| = \begin{vmatrix} z & S_k & 1 \\ 1 & -\frac{1}{\omega}(1-S_k) & 0 \\ 1 & \frac{1}{\omega}S_k & -\eta \end{vmatrix} = \frac{1}{\omega}\left[(1-S_k)(1+z\eta) + S_k\right] + \eta S_k > 0$$

we arrive at the solutions (13), (14) and (15).

Proof of Lemma 2

(i) We have $\widehat{\left(\frac{c}{e}\right)} = \hat{c} - \hat{e}$. Logarithmic time differentiation of (4) and combining this with (5) noting that $\hat{e}(1, p) = \frac{pe_2}{e}\hat{p} = s(p)\hat{p}$ we obtain,

(A11) $\hat{c} - \hat{e} = \frac{1}{a}\left[M - s(p)\hat{p}\right]$.

(ii) From (A11) it follows that real consumption grows over time if $\hat{p} < \frac{M}{s(p)}$. From (13) we can decompose $\hat{p}$ as follows,

$\hat{p} \equiv \hat{p}_0 + \hat{p}_b$, where $\hat{p}_0 = \dfrac{\frac{M}{\omega}(1 - S_k)\left[\frac{\eta}{a} + 1\right]}{|W|}$ and $\hat{p}_b = \dfrac{-\frac{(1 - S_k)}{\omega}\zeta(\eta + 1)}{|W|}$. Then we find that a sufficient condition for $\hat{p} < \frac{M}{s(p)}$ to hold is,

(A12) $\hat{p}_0 = \dfrac{(1/\omega)M(1 - S_k)\left[(\eta / a) + 1\right]}{(1/\omega)\left[(1 - S_k)(1 + z\eta) + S_k\right] + \eta S_k} < \dfrac{M}{s(p)}$

Rearranging (A12) we have,

(A13) $(1 - S_k)\left(\frac{\eta}{a} + 1\right)s(p) < \left[(1 - S_k)(1 + z\eta)\right] + S_k + \eta S_k\omega$

Since, $(S_k + \eta S_k\omega) > 0$ and $z \equiv \frac{s(p)}{a} + (1 - s(p))\sigma$, (A13) is satisfied if the following inequality holds,

(A14) $\dfrac{\eta s(p)}{a} + s(p) < 1 + \dfrac{\eta s(p)}{a} + (1 - s(p))\sigma\eta$, or, equivalently if $0 < (1 - s(p))(1 + \sigma\eta)$, which is always true for $0 < s(p) < 1$. Thus, we have $\hat{p} < (M / s(p))$ at any finite point of time and for all finite σ and ω. That is, real consumption growth is positive along the equilibrium dynamic path.

(iii) If $\omega > 1$ then $\lim_{t\to\infty} S_k = 1$, which implies that $\lim_{t\to\infty} \hat{p} = 0$;also, if $\sigma > 1$ then $\lim_{t\to\infty} s = 0$. In either case we have that $s(p)\hat{p}$ converges to zero. Thus, from (A11) it follows that in either case the growth of real consumption converges from below towards M / a; that is $\lim_{t\to\infty} \left(\hat{\frac{c}{e}}\right) = M / a$.

(iv) If both $\omega < 1$ and $\sigma < 1$ then $\lim_{t\to\infty} S_k = 0$ and $\lim_{t\to\infty} s = 1$. This implies that $\lim_{t\to\infty} \hat{p} = \frac{(1+\eta/a) - (1+\eta)\zeta}{1 + z\eta}$. But since $\lim_{t\to\infty} s = 1$ we have that $\lim_{t\to\infty} z = 1/a$. Therefore, we have that $\lim_{t\to\infty} \hat{p} = M - \frac{(1+\eta)\zeta}{1+\eta/a}$. Thus, using this expression in (A11) and considering the fact that $\lim_{t\to\infty} s = 1$ we have,

$$\lim_{t\to\infty} \left(\hat{\frac{c}{e}}\right) = \frac{1+\eta}{a+\eta}\zeta$$

Finally, we show that $s\hat{p}$ is increasing over time, meaning that $\left(\hat{\frac{c}{e}}\right)$ converges towards the above limit from above. Using the definitions of $|W|$ and z, and equation (13) it follows that we can write,

$$s\hat{p} = \frac{(1+\eta)\left[\frac{1+\eta/a}{1+\eta} M - \zeta\right]}{1 + \frac{s\eta}{a} + \frac{(1-s)}{s}\sigma\eta + \frac{S_k}{1-S_k}(1+\eta\omega)}$$

Clearly, this expression is increasing in s and decreasing in S_k. If $\sigma < 1$ it follows that s is increasing over time as p increases. Also, since k_d / bx increases over time, the fact that

$\omega < 1$ implies that S_k is falling. Thus, along the equilibrium growth path $s\hat{p}$ is increasing. Hence, we have that $\widehat{\left(\frac{c}{e}\right)} = \frac{1}{a}\left[M - s(p)\hat{p}\right]$ must be falling over time. That is, the rate of growth of real consumption converges to a positive rate $\frac{1+\eta}{a+\eta}\zeta$ from above; If $\sigma < 1$ and $\omega < 1$ then rate of economic growth is declining over time.

<u>Proof of corollary 1:</u>

If $\sigma > \frac{1}{a} > 1$ and in the case when $\omega < 1$ then $\lim_{t\to\infty} \hat{x}$ is given by Equation (16). Hence, it follows that using Assumption 2, $\lim_{t\to\infty} \hat{x} < 0$ if and only if $\zeta < \frac{M}{a}\left(\frac{a\sigma - 1}{\sigma - 1}\right)$.

<u>Proof of corollary 2:</u>

When $\omega > 1$, S_k converges to 1, and $\hat{p}$ remains positive, while $s(p)$ converges to 0 if $\sigma > 1$. Then Equation (17) applies. Hence, from (17) it follows that $\lim_{t\to\infty} \hat{x}$<0 if and only if

(A15) $$\zeta < \left(\frac{\omega - 1/a}{\omega - 1}\right)M$$

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